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The History of Euclidean Algorithm

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Abstract: The Euclidean Algorithm is one of the oldest numerical algorithms still in use today. Attributed to ancient Greek mathematician Euclid in his book "Elements" written approximately 300 BC, the algorithm serves as an effective method for finding the greatest common divisor of two whole numbers. This article discusses about Euclidean Algorithm.

Key words: Euclidean Algorithm, math sums, irrational numbers, natural numbers, etc.

The greatest common divisor (GCD) of two whole numbers is the largest natural number that divides evenly into both without a remainder. We'll begin with the formal definition and then work an example. If mathematical notation makes your eyes glaze over, hang in there! It's actually pretty straightforward when you see the example. In the initial setup, the values for a-0 and b are the two values we want to find the GCD of, a-o being the larger of the two. The goal of each step is to find the quotient q and remainder r that makes the equation true. The Euclidean Algorithm is a k-step iterative process that ends when the remainder is zero. (In other words, you keep going until there's no remainder.) The GCD will be the last non-zero remainder. Euclidean algorithm, procedure for finding the greatest common divisor (GCD) of two numbers, described by the Greek mathematician Euclid in his Elements (c. 300 BC). The method is computationally efficient and, with minor modifications, is still used by computers.

The algorithm involves successively dividing and calculating remainders; it is best illustrated by example. For instance, to find the GCD of 56 and 12, first divide 56 by 12 and note that the quotient is 4 and the remainder is 8. This can be expressed as $56 = 4 \times 12 + 8$. Now take the divisor (12), divide it by the remainder (8), and write the result as $12 = 1 \times 8 + 4$. Continuing in this manner, take the previous divisor (8), divide it by the previous remainder (4), and write the result as $8 = 2 \times 4 + 0$. Since the remainder is now 0, the process has finished and the last nonzero remainder, in this case 4, is the GCD. The Euclidean algorithm is useful for reducing a common fraction to lowest terms. For example, the algorithm will show that the GCD of 765 and 714 is 51, and therefore $765/714 = 15/14$. It also has a number of uses in more advanced mathematics. For example, it is the basic tool used to find integer solutions to linear equations $ax + by = c$, where a, b, and c are integers. The algorithm also provides, as the successive quotients obtained from the division process, the integers a, b, …, f needed for the expansion of a fraction p/q as a continued fraction:

$$a + 1/(b + 1/(c + 1/(d \dots + 1/f).$$

The Euclidean Algorithm is truly fundamental to many other algorithms throughout the history of computer science and will definitely be used again later. At least to me, it's amazing how such an ancient algorithm can still have modern use and appeal. That said, there are still other algorithms out there that can find the greatest common divisor of two numbers that are arguably better in certain cases than the Euclidean algorithm, but the fact that we are discussing Euclid two millennia after his death shows how timeless and universal mathematics truly is. I think that's pretty cool. Computer science is (almost by definition) a science about computers -- a device first conceptualized in the 1800's. Computers have become so revolutionary, that it is difficult to think of our lives today without them. That said, algorithms are much older and have existed in the world for millennia. Incredibly, a few of the algorithms created before the Common Era (AD) are still in use today. One such algorithm was first described in Euclid's Elements (~ 300 BC) and has come to be known as the Euclidean Algorithm. More than two millennia ago Euclid (circa 300 BCE) described a method for computing the "greatest common measure" of two "numbers", and today we name our modern iterative algorithm for calculating the greatest common divisor of two numbers after him. Here we introduce and provide for instructors a student project based on Euclid's original source, designed for a course in introductory discrete mathematics or computer science. The similarities and differences between our modern algorithms and what Euclid presented and proved are extremely fertile ground for initial student learning about algorithms and proofs. For instance, as shown below, Euclid adopted a highly geometric description of his numbers as lengths. Euclid's presentation also naturally sets the stage, if desired, to extend the project by comparing and contrasting with a modern day recursive, as opposed to iterative, formulation of the algorithm, as it is often presented to computer science students. And the highly contrasting proofs of correctness in these two very different settings can be explored.

The project can be used to provide a first introduction to the notion of "computation method" or "algorithm" and to explore concepts like iteration and the efficacy of mathematical induction as a method of proof, thereby covering a number of typical course topics. The project can even be used to introduce induction. With this project students can develop their skill at creating proofs in a highly authentic and motivated context, but just as importantly they can experience the evolution of what is accepted as a valid proof or a well-described algorithm. Students will learn that the method presented by Euclid to compute the greatest common divisor and the proof of its correctness that he provided would not be formally accepted today. Students will also experience, however, that Euclid was somehow able to convey the ideas behind his method and proof in such a way that they can reform Euclid's writing into a modern algorithm and proof of correctness. In this way, the project provides students not only with a strong sense of connection to the past, but also serious practice with subtle issues about the nature of adequate mathematical formulation and proof today. Students can work productively in groups on this project, with group or individual writeups. They will need substantial guidance with the optional exercises near the end of the project, which formalize the algorithm in wholly modern terms and prove correctness using finite induction. In any case, the instructor should always work through all details before assigning any student work. The issue of "unit" versus "number" will provide grist for substantial class discussion and careful attention to detail in interpreting Euclid's analysis of his algorithm. It seems clear that by "unit" Euclid meant what we today call the number "one", but that to him it was not a number. Why this was the case for Euclid, and how it played out in his writings, is rich material for critical consideration when studying Euclid.

In mathematics, the Euclidean algorithm,[note 1] or Euclid's algorithm, is an efficient method for computing the greatest common divisor (GCD) of two integers (numbers), the largest number that divides them both without a remainder. It is named after the ancient Greek mathematician Euclid, who first described it in his Elements (c. 300 BC). It is an example of an algorithm, a step-by-step procedure for performing a calculation according to well-defined rules, and is one of the oldest algorithms in common use. It can be used to reduce fractions to their simplest form, and is a part of many other number-theoretic and cryptographic calculations. Euclid's method for finding the greatest common divisor (GCD) of two starting lengths BA and DC, both defined to be multiples of a common "unit" length. The length DC being shorter, it is used to "measure" BA, but only once because remainder EA is less than DC. EA now measures (twice) the shorter length DC, with remainder FC shorter than EA. Then FC measures (three times) length EA. Because there is no remainder, the process ends with FC being the GCD. On the right Nicomachus's example with numbers 49 and 21 resulting in their GCD of 7 (derived from Heath 1908:300).

The Euclidean algorithm is based on the principle that the greatest common divisor of two numbers does not change if the larger number is replaced by its difference with the smaller number. For example, 21 is the GCD of 252 and 105 (as $252 = 21 \times 12$ and $105 = 21 \times 5$), and the same number 21 is also the GCD of 105 and $252 - 105 = 147$. Since this replacement reduces the larger of the two numbers, repeating this process gives successively smaller pairs of numbers until the two numbers become equal. When that occurs, they are the GCD of the original two numbers. By reversing the steps or using the extended Euclidean algorithm, the GCD can be expressed as a linear combination of the two original numbers, that is the sum of the two numbers, each multiplied by an integer (for example, $21 = 5 \times 105 + (-2) \times 252$. The fact that the GCD can always be expressed in this way is known as Bézout's identity.

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